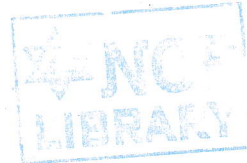


TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2082 Shrawan

Exam.	Back		
Level	BE	Full Marks	80
Programme	BCE	Pass Marks	32
Year / Part	IV / II	Time	3 hrs.

Subject: - Time Series Analysis (*Elective III*)(CE78505)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt **All** questions.
- ✓ The figures in the margin indicate **Full Marks**.
- ✓ Assume suitable data if necessary.



1. Describe the conceptual representation of the precipitation-runoff process within a watershed system with a neat sketch. [4]
2. How can the trend of time series data be detected using (i) a regression test (ii) spearman's rank correlation test (iii) the Mann-Kendall test? [2+2+2]
3. If two random variables X and Y have the joint pdf given by [1+1+2]

$$f(x,y) = (3x+y)/4 \text{ for } 0 < x < 1, 0 < y < 2$$

$$= 0 \text{ elsewhere}$$

Find the marginal density of X and the marginal density of Y. Also, verify whether two random variables are independent or not.
4. The probability distribution function of a random variable is $f(x) = \lambda e^{-\lambda x}$ for $x > 0$. Find the parameter λ of the distribution by the method of moments and the method of maximum likelihood. [3+3]
5. Explain the concepts of correlation and regression, highlighting the key differences between them. [2+2]
6. Explain the various types of time series that illustrate deterministic processes, providing appropriate examples. [8]
7. Derive equations that describe the population autocorrelation coefficients for the AR (m) model. Also, establish the relationship between the variance of the AR (m) process and that of the independent process. [6+5]
8. Classify the $(X_t - \mu) = (X_{t-1} - \mu) + e_t + 0.5e_{t-1}$ process as either AR, MA or ARMA. Verify whether it is stationary or not. [1+3]
9. The annual flow values (m^3/s) of a river over 10 years are 500, 420, 490, 550, 430, 570, 510, 540, 470 and 480. If an AR (1) model is considered suitable, determine its parameter. If an MA(1) model is applied to the data, what would its parameter be? [10+2]
10. The mean monthly flow (m^3/s) of a river varies as follows: 65, 45, 35, 25, 20, 240, 780, 1550, 1605, 930, 210 and 85. Fit a Fourier series that captures more than 90% of the variance and plot the line spectrum. [10+2]
11. Describe the different techniques used to generate normal random numbers. [9]

Exam.	Regular	
Level	BE	Full Marks 80
Programme	BCE	Pass Marks 32
Year / Part	IV/ II	Time 3 hrs.

Subject: - Time Series Analysis (Elective III) (CE 78505)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ The figures in the margin indicate Full Marks.
- ✓ Assume suitable data if necessary.



1. Define the following terms in the context of time series: Model, Population, Sample, Random variable, Autocorrelation function and Ergodicity. [6]
2. Describe different types of hydrological time series. [7]
3. The probability distribution function of a monsoon storm is as given below. [6]

$$f(x) = kx^2 \text{ for } 0 \leq x \leq 6h$$

$$f(x) = k(12-x^2) \text{ for } 6 \leq x \leq 12h$$

$$f(x) = 0 \text{ elsewhere}$$
 Find the value of k . Also, find the mean and standard deviation.
4. The probability distribution function of a random variable is $f(x) = a e^{-ax}$ for $x > 0$. Find the parameter a of the distribution by the method of moments and the method of maximum likelihood. [3+3]
5. Using the annual run-off data of 40 years, the first 10 autocorrelation coefficients have been calculated and are presented below. Perform a test for independence on the series at a 5% level of significance. Also, suggest a suitable model to describe the series. [6+2]

Lag k (years)	1	2	3	4	5	6	7	8	9	10
r_k	0.62	0.40	0.18	0.14	0.03	-0.02	0.07	0.01	0.00	-0.02
6. Derive equations representing population autocorrelation coefficients of the AR (m) model. Also, develop a relationship of the variance between the AR (m) process and the independent process. [6+5]
7. The annual river flow values (m^3/s) over 10 years are: 500, 420, 490, 550, 430, 570, 510, 540, 470, 480. Assuming AR(1) model is a good fit, determine its parameter. [8]
8. Classify the following model as AR, MA, or ARMA. Also, find first six serial correlation coefficients. $(X_t - \mu) = 0.5(X_{t-1} - \mu) + e_t$ [1+2]
9. What is the Akaike Information Criterion (AIC)? How is it used in model selection? [2+1]
10. Define spectral analysis and line spectrum. [1+1]
11. Following are the mean monthly mean flows (m^3/s): 65, 45, 35, 25, 20, 240, 780, 1550, 1605, 930, 210, 85. Fit a Fourier series capturing more than 90% variance and plot a line spectrum. [8+2]
12. Generate 12 uniformly distributed random numbers in the range 0 to 0.999 using the mixed congruential generator, with parameters: Multiplier = 316, Increment = 56387, Modulus = 10^5 , Seed = 42567. Also, obtain two normal random numbers from the sequence using the central limit theorem method. [6+4]

Exam.	Regular		
Level	BE	Full Marks	80
Programme	BCE	Pass Marks	32
Year / Part	IV / II	Time	3 hrs.

Subject: - Time series Analysis (Elective III) (CE78505)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt **All** questions.
- ✓ The figures in the margin indicate **Full Marks**.
- ✓ **Necessary tables are attached herewith.**
- ✓ Assume suitable data if necessary.



1. Differentiate between deterministic and random process. List the applications of time series modeling in water resources engineering. [2+2]
2. Define the following terms: Model, White Noise and Realization. [1+1+1]
3. What do you mean by stationary and non-stationary time series? Explain the method for assessing the stationarity of time series data through the split record test. [1+4]
4. The annual peak discharge (m^3/s) of a river basin for a period of 22 years is given below: [8]
 $18.63, 20.91, 15.75, 15.46, 14.50, 13.05, 17.24, 17.89, 13.84, 13.14, 15.38, 11.67, 13.55, 20.44, 29.32, 20.08, 16.35, 14.72, 10.17, 22.95, 22.89,$ and 26.52
 Test the goodness of fit of normal distribution to the observed data at 5% significance level by Chi-squared test. Utilise a class interval size of 5
5. State and prove Chebyshev's inequality. What are its applications? [4+2]
6. The first 6 autocorrelation coefficients computed from 32 years of annual flow record at a river site are: $0.430, -0.077, -0.457, -0.370, 0.057$ and 0.423 . Test the flow series for independence at 95% confidence level. [6]
7. Starting from Yule-Walker equation for $AR(m)$ model, obtain the parameters of $AR(1)$ and $AR(2)$ model. [5]
8. Classify the following model. Check whether it is stationary or not. Also, find first five serial correlation coefficients. $(x_t - \mu) = 0.5(x_{t-1} - \mu) + e_t$. [1+2+2]
9. Assuming that an $AR(2)$ model provides a stationary fit to annual flow of a river whose mean and standard deviation are estimated to be $510 Mm^3$ and $120 Mm^3$, respectively. The first and second correlation coefficients are 0.352 and -0.023 respectively. Generate the annual flow for three years using the chain of normal random variates to be $0.2518, -0.6224$ and -0.5738 . [10]
10. A hydrologic variable is characterized by an $AR(m)$ model with $\sigma_x = 4.85, r_1 = 0.430$ and $r_2 = -0.077$. Determine the optimal parameters of the $AR(m)$ model using the Akaike Information Criteria. [6]
11. The mean monthly flows (m^3/s) of a river are $14.69, 48.16, 60.05, 41.98, 17.55, 7.13, 4.35, 3.38, 2.89, 2.78, 2.80,$ and 3.94 . Apply the Fourier series to capture more than 90% of the variance and plot a line spectrum. [10+2]
12. Generate a chain of 12 uniformly distributed random numbers within the range 0 to 0.999 using the mixed congruential generator. Take multiplier = 316, increment = 56737, modulus = 10^5 and seed = 42675. Also, obtain two normal random numbers from the sequence of the generated uniform random numbers using the central limit theorem method. [6+4]

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Examination Control Division
2078 Chaitra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	BCE	Pass Marks	32
Year / Part	IV / II	Time	3 hrs.

Subject: - Time Series Analysis (Elective III) (CE78505)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ The figures in the margin indicate Full Marks.
- ✓ Assume suitable data if necessary.



1. Define the terms model, population, sample, random variable, realization, normalization, ensemble and ergodic. [4]
2. a) Enlist different types of hydrological time series data. [3]
b) Give the importance of trend in time series. [3]
3. The probability distribution function of a monsoon storm is as given below. [6]

$$f(x) = kx^2 \text{ for } 0 \leq x \leq 6h$$

$$f(x) = k(12-x^2) \text{ for } 6 \leq x \leq 12h$$

$$f(x) = 0 \text{ elsewhere}$$

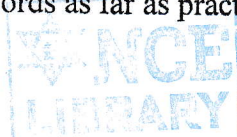
Find the value of k. Also, find the mean and standard deviation.
4. State and prove Chebyshev's inequality principle. Explain limitations of Chebyshev's theory. [2+3+1]
5. The first 9 autocorrelation coefficients computed from 150 years of annual flow at a river site are: 0.425, 0.005, -0.095, -0.05, -0.042, -0.01, -0.018, -0.016, -0.055. Test the series for independence at 5% level of significance. Also suggest a suitable model for the series. [8]
6. Discuss general properties of normal distribution. [3]
7. Describe the methods for the determination of parameters of the time series models. [6]
8. Write the basic equation of MA, MA (2), ARMA (1, 1) process. [3]
9. Assuming that an AR (2) model provides a satisfactory fit to annual flow of a river whose mean and standard deviation are estimated to be 515 Mm^3 and 120 Mm^3 . The first and second serial correlation coefficients are 0.458 and -0.004 respectively. Generate the annual flows for 3 years using the chain of normal random numbers as 0.2659, -0.8176 and -0.6789. [10]
10. For MA (2) process, following data are given: $r_1 = 0.388$ and $r_2 = 0.259$. Compute the parameters β_1 and β_2 . [5]
11. What is spectral analysis? Why is it important in hydrological data analysis? [3]
12. A process is given by $(X_t - \mu) = 1.9((X_{t-1} - \mu) - 0.3(X_{t-2} - \mu)) + e_t$. Classify the process as AR, MA or ARMA. Is this process stationary? Determine the variance of X_t given the variance of e_t as 1.0. Also obtain first five serial correlation coefficients. [2+2+2+2]
13. Generate a sequence of 12 uniformly distributed random numbers in the range of 0 to 0.999 using the mixed congruential, technique. Take multiplier = 361, increment = 56377, modulus = 10^5 and seed = 42675. And obtain the sequence of normal random numbers using Box - Muller method by using the previous sequence of uniform random numbers. [12]

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Examination Control Division
2077 Chaitra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	BCE	Pass Marks	32
Year / Part	IV / II	Time	3 hrs.

Subject: - Time Series Analysis (Elective III) (CE78505)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ The figures in the margin indicate Full Marks.
- ✓ Assume suitable data if necessary.



1. Explain the following terms: Times series, white noise, realization and ergodic property. [8]
2. a) What do you mean by persistency in a time series? Differentiate between stationary and non-stationary time series. [4]
- b) For the following time series, examine whether there is trend in the data or not using Spearman's rank correlation test. [4]
 10.2, 12.4, 14.8, 15, 11.2, 14.3, 18.4, 18
 Take value of Tabulated T for the given significance level as 2.4.
3. a) The probability density function of a random variable is given by $f(x) = \lambda e^{-\lambda x}$ for $x > 0$. Determine λ of the distribution by (i) Methods of moment (ii) Methods of maximum like hood. [6]
- b) Why do you need to perform Tests for Goodness of Fit in time series analysis? Explain about Chi-square test for Goodness of Fit. [2+4]
4. a) What is correlogram? Explain the nature of correlogram plot for: [1+4]
 - (i) Stationary time series
 - (ii) Non stationary time series
 - (iii) random series
 - (iv) periodic series
- b) Mean daily discharge (m^3/s) of a river for 10 days is given below: [7]
 45, 42, 65, 73, 102, 121, 149, 225, 450, 375. Compute autocorrelation coefficient for first and second year lag using the simplified form of equation.
5. a) Explain about the procedure which are followed for fitting the time series model. [6]
- b) Obtain the parameter of AR(2) model from Yule-Walker equation. Also discuss the condition of stationarity and the nature of ACF of AR(2) model with sketches. [6]
- c) Classify the following process as AR, MA or ARMA. Verify whether they are stationary. $(X_t - \mu) = 0.3(X_{t-1} - \mu) + e_t + 0.7e_{t-1}$. Also determine the five serial correlation coefficients. [3+5]
6. a) Explain the Thomas-Fiering model of generating seasonal flows. [10]
- b) What is the difference between correlogram and periodogram? [2]
7. Generate a chain of 8 uniformly distributed random numbers in the range 0 to 0.999 using the mixed congruential generator. Take multiplier = 361, increment = 56377, modulus = 10^5 and seed = 42765. [8]

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INSTITUTE OF ENGINEERING
Examination Control Division
2076 Bhadra

Exam.	Regular / Back		
Level	BE	Full Marks	80
Programme	BCE	Pass Marks	32
Year / Part	IV / II	Time	3 hrs.

Subject: - Time Series Analysis (Elective III) (CE 78505)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ The figures in the margin indicate Full Marks.
- ✓ Assume suitable data if necessary.

1. Describe time series modeling with examples. List the applications of time series modeling in water resources engineering. [2+3]
2. a) Enlist different types of hydrological time series data. Describe any two in detail. [2+4]
b) What is trend in time series? Discuss two methods for the detection of trend. [4]
3. a) The joint probability density function given as:
 $f(x,y) = 0.5*(2x+7y)$ for $0 < x < 1, 0 < y < 1$
 $= 0$, elsewhere
 Find the marginal function of X and Y. Are these random variables independent? [2+2]
 b) State and prove Chebyshev's inequality principle. [2+4]
4. The first 9 autocorrelation coefficients computed from 150 years of annual flow at a river site are : 0.461, 0.005, -0.095, -0.05, -0.048, -0.01, -0.018, -0.016, -0.055. Test the series for independence at 90% confidence level. Also suggest a suitable model for the series. [8]
5. a) Assuming that an AR(2) model provides a satisfactory fit to annual flow of a river whose mean and standard deviation are estimated to be 525 Mm^3 and 140 Mm^3 . The first and second serial correlation coefficients are 0.458 and -0.004 respectively. Generate the annual flows for 3 years using the chain of normal random numbers as 0.2619, -0.7176 and -0.5749. [10]
 b) A process is given by $(X_t - \mu) = 1.5(X_{t-1} - \mu) - 0.5(X_{t-2} - \mu) + e_t$. Classify the process as AR, MA or ARMA. Is this process stationary? Determine the variance of X_t given the variance of e_t as 1.0. Also obtain first five serial correlation coefficients. [2+2+2+5]
 c) Starting from basic equation of MA(m) obtain the parameters of MA(1) and MA(2). [6]
6. Generate synthetic seasonal flows for 4 seasons for a year using the following statistical parameters and Thomas -Fiering model. [10]

Season	Mean flow (m^3/s)	Standard deviation	Correlation with previous season	$Z_t = N(0,1)$
1	10	8.8	0.63	0.25
2	20	18	0.62	-1.71
3	60	57	0.20	0.31
4	9	8	0.05	0.11

7. Generate a chain of 8 uniformly distributed random numbers in the range 0 to 0.99999 using the mixed congruential generator. Take appropriate multiplier, increment, modulus and seed. And also obtain the sequence of normal random numbers using Box-Muller Method by using the previous sequence of uniform random numbers. [6+4]

Exam.	Regular		
Level	BE	Full Marks	80
Programme	BCE	Pass Marks	32
Year / Part	IV / II	Time	3 hrs.

Subject: - Time Series Analysis (Elective III) (CE78505)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt **All** questions.
- ✓ The figures in the margin indicate **Full Marks**.
- ✓ Assume suitable data if necessary.

1. Differentiate between deterministic and random process. And also list the applications of time series modeling in water resources engineering. [3+3]

2. Enlist the different types of hydrological time series data. Describe any two in detail. [2+4]

3. a) The joint probability density function given as;

$$f(x, y) = \frac{2}{3}(x + 2y) \text{ for } 0 < x < 1, 0 < y < 1$$

$$= 0, \text{ elsewhere}$$

Find the marginal function of X and Y. Are these random variables independent? [3+3]

b) The annual runoff of a stream has a mean of 256 and standard deviation of 190 m³/s respectively. Find the ranges within which the runoff will lie with a probability of at least 0.5. Also, compare the Probability with the lower bound provided by Chebyshev's inequality. For Normal distribution, take z = 0.0785 for F(Z) = -1.414 and z = 0.921 for F(Z) = 1.414. [3+3]

4. a) What is autocorrelation? How is it useful for the investigation of independent process? [2+2]

b) Consider the following set of data: {23.32, 32.33, 32.88, 28.98, 33.16, 26.33, 29.88, 32.69, 18.98, 21.23, 26.66 and 29.89}. Calculate the lag-one, lag-two and lag-three sample autocorrelation of the time series. [2+2+2]

5. a) A process is given by $(X_t - \mu) = 1.2(X_{t-1} - \mu) - 0.5(X_{t-2} - \mu) + e_t$. Classify the process as AR, MA or ARMA. Is this process stationary? Determine the variance of X_t given the variance of e_t as 1.0. Also obtain the five serial correlation coefficients. [2+2+2+4]

b) The first serial correlation coefficient of MA (1) process is 0.4. Determine the parameter of the process. [4]

c) Assuming an ARMA(1, 1) model is a good fit to describe the normally distributed annual flows of a stream whose mean, standard deviation, first and second serial correlation coefficients of observed annual flows of a stream are estimated to be 1210 Mm³, 570 Mm³, 0.535 and 0.463 respectively. Generate 3 annual flows. The sequence of 3 standard normal random numbers may be taken as + 1.123, - 0.821 and - 0.342. [10]

6. Generate synthetic seasonal flows for 4 seasons for a year using the following statistical parameters and Thomas-Fiering model. [10]

Season	Mean flow (m3/s)	Standard deviation	Correlation with previous season	$Z_t: N(0,1)$
1	10	8.8	0.63	0.25
2	20	18	0.62	-1.71
3	60	57	0.20	0.31
4	9	8	0.05	0.11

7. a) Generate a chain of 6 uniformly distributed random numbers in the range 0 to 0.999 using the mixed congruential generator. Take multiplier = 361, increment = 56377, modulus = 10⁵ and seed = 42765. [6]

b) Explain the central limit theorem method for the generation of normally distributed random numbers. [6]

Exam.	Regular		
Level	BE	Full Marks	80
Programme	BCE	Pass Marks	32
Year / Part	IV / II	Time	3 hrs.

Subject: - Time Series Analysis (Elective III) (CE78505)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt **All** questions.
- ✓ The figures in the margin indicate **Full Marks**.
- ✓ Assume suitable data if necessary.

1. Explain time series data with examples. Discuss the applications of time series analysis in water resources engineering. [2+4]
2. Differentiate between stationary and non-stationary time series. Explain the split record test for detecting stationarity in mean and variance of a time series data. [3+5]
3. (a) The duration of a monsoon storm is a random variable whose pdf is given by

$$f(x) = kx^2 \text{ for } 0 \leq x \leq 3h$$

$$f(x) = k(6 - x)^2 \text{ for } 3 \leq x \leq 7h$$

$$f(x) = 0 \text{ elsewhere}$$
[2+2+2]

Find the value of k. Determine the corresponding CDF. What is the probability that the duration of the storm is less than 2h?
- (b) Explain marginal probability. Rainfall data considered as random variable has a mean of 20 and variance of 16 and unknown probability distribution. Find the least probability with which it lies between 12 and 28. [2+4]
4. (a) Derive Yule-Walker's equations for Autoregressive (AR) model. What is the application of these equations in time series modeling? [8]
- (b) The first 10 autocorrelation coefficients computed from 100 years of annual flow record at a river site are: 0.15, 0.09, 0.02, -0.12, -0.11, 0.05, 0.07, -0.1, -0.08, 0.02 [8]

Making a plot of correlogram, test the flow series for independence at 95% confidence level.
5. (a) Following is a form of a time series model fitted to a data set.

$$(X_t - \mu) = 0.9(X_{t-1} - \mu) + e_t + 0.4e_{t-1}$$

What is the type of the above model? Also mention the order of the model. Verify whether the model is stationary. [4]
- (b) Following are the annual flow values (m^3/s) of a river for 10 years: 290, 248, 323, 292, 294, 236, 225, 263, 213, 190 [10+2]

Assuming AR(1) model to be good fit for the above data, find out the parameter of the model. Instead of AR(1) model, if you want to select MA(1) model for the data, what will be the value of its parameter?
6. (a) Explain the concept of spectral analysis. [2]
- (b) Generate synthetic seasonal flows for 4 seasons for a year using the following statistical parameters by using Thomas-Fiering model. [12]

Season	Mean flow (m^3/s)	Standard deviation	Correlation with previous season	$Z_t: N(0,1)$
1	13	9	0.67	0.35
2	25	12	0.61	-1.52
3	54	50	0.18	0.24
4	10	7	0.07	0.15

Exam.	Back		
Level	BE	Full Marks	80
Programme	BCE	Pass Marks	32
Year / Part	IV / II	Time	3 hrs.

Subject: - Time Series Analysis (*Elective III*) (CE78505)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ The figures in the margin indicate Full Marks.
- ✓ Assume suitable data if necessary.

1. How is time series analysis linked to the physics of hydrological processes? Explain by giving an example and appropriate expressions. [6]
2. Discuss homogeneity and seasonality in time series. Describe any one method to detect the randomness in a time series. [2+4]
3. (a) Explain the terms with appropriate expressions: Conditional distribution, marginal distribution and derived distribution. [2+2+2]
- (b) The probability density function of daily rainfall is given as [6]

$\text{Prob}[X=0]=0.2$
 $f(x) = \frac{x}{5}$ for $0 < x < 1.0$
 $f(x) = 1.2 - \frac{x}{5}$ for $1.0 < x < 2.0$

Is this a proper probability density function? If yes, what is $P[X > 0.75]$ and what is $P[X > 0.75 / X \neq 0]$?
4. (a) Explain the methods applied for estimating parameters of time series models. [6]
- (b) Autocorrelation coefficient from lag 1 to lag 10 of a variable is given below: [4]

0.62 0.13 0.09 -0.11 0.08 -0.06 0.16 -0.06 0.10 -0.073

Making a plot of ACF, suggest an appropriate order of AR model.
- (c) Identify the type of model and compute the first five autocorrelation coefficient for the process represented by following expression: [6]

$(X_t - \mu) = e_t + 1.1e_{t-1} + 0.3e_{t-2}$
5. (a) The first and second serial correlation coefficients of observed annual discharge are 0.7 and 0.5 respectively. Assuming AR (2) model is a good fit, find the parameters of the model. [4]
- (b) Fit an AR(1) model for the following annual rainfall data in mm: [10]

180, 150, 170, 145, 130, 124
- (c) What is an MA model? [2]
6. (a) Assuming an ARMA(1,1) model is a good fit to describe the normally distributed annual flows of a stream whose mean, standard deviation, first and second serial correlation coefficients of observed annual flows of a stream are estimated to be 1125 Mm³, 536 Mm³, 0.64 and 0.41 respectively. Generate 3 annual flows. The chain of 3 standard normal random numbers may be taken as 1.024, 1.057 and 0.936. [14]
- (b) Mention the salient features of ARMA(1,1) model. [2]
7. Describe the methods applied to generate uniformly distributed random numbers. [8]

6 E TRIBHUVAN UNIVERSITY
 INSTITUTE OF ENGINEERING
Examination Control Division
 2073 Bhadra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	BCE	Pass Marks	32
Year / Part	IV / II	Time	3 hrs.

Subject: - Time Series Analysis (Elective III) (CE78505)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ The figures in the margin indicate Full Marks.
- ✓ Assume suitable data if necessary.

1. What is the time series modeling and List the applications of time series modeling in water resources engineering. [6]
2. a) Differentiate the following: [4]
 - (i) Stationary and non-stationary time series
 - (ii) Trend and periodicity
- b) Discuss any two methods for the detection of trend. [4]
3. a) Where does the stochasticity come from in a hydrological process? [2]
- b) State and prove the Chebyshev's inequality. [2+4]
- c) The pdf of normal distribution is given by $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$. Find the parameters μ and σ by method of maximum likelihood. [4]
4. a) What is cross-correlation and auto correlation? How is correlation different from regression? [2+2+2]
- b) From a data set of 30 years flow, the auto correlation coefficient $r_k = [0.66, 0.31, 0.01, 0.0, 0.01, 0.0]$ for $k = 1$ to 6. Plot the ACF (not necessary to the scale) and state how many lag times would you consider to fit a time series model to the data. [6]
5. The \bar{x}, s_x, r_1, r_2 of the observed annual flows of a stream are estimated as $875 \text{ Mm}^3, 262 \text{ Mm}^3, 0.8$ and 0.46 respectively. Generate a sequence of 3 annual flows assuming that the flows are normally distributed and taking the chain of three uniform random number as $0.3781, 0.63702, 0.98343$. Use AR (2) model. [14]
6. Write down Thomas-Fiering model for generation of seasonal flows. Explain the significance of each term in the model and explain clearly how the non-stationarity and cyclicity of the seasonal flows are preserved. [14]
7. a) Generate a chain of 5 uniformly distributed random numbers using mid-square and mid-product techniques. Assume the seed. [6]
- b) Explain how do you fit a time series using spectral analysis. [8]

Exam.	New Back (2066 & Later Batch)		
Level	BE	Full Marks	80
Programme	BCE	Pass Marks	32
Year / Part	IV / II	Time	3 hrs.

Subject: - Time Series Analysis (*Elective III*) (CE78505)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ The figures in the margin indicate Full Marks.
- ✓ Assume suitable data if necessary.

1. Discuss the time series model. What are the advantages and limitations of such model? [6]
2. Explain trend, periodicity and jump in time series data with examples and sketches. [6]
3. a) Describe conditional distribution, marginal distribution and derived distribution. [6]
 - b) The annual runoff of a stream has a mean of 510 and standard deviation of $90\text{m}^3/\text{s}$ respectively. Find the ranges within which the runoff will lie with a probability of at least 0.5. Use Chebyshev's inequality. [6]
4. a) Define autocorrelation and list the equations used for its computation. How can you use the autocorrelation analysis in the time series analysis? [4+4]
 - b) Autocorrelation coefficient from lag 1 to lag 5 of a variable is given below.
 0.5 0.3 0.0 0.0 0.0
 Making a plot of correlogram, suggest an appropriate order of MA model. [6]
 - c) A hydrologic variable is described by an AR(1) model with mean = 120, variance = 950, first serial correlation coefficient = 0.5. Compute the standard deviation of the random process. [4]
5. a) Discuss the nature of autocorrelation function of AR(1) and AR(2) model with sketches. [4]
 - b) The mean, standard deviation, first and second serial correlation coefficients of observed annual flows of a stream are estimated as 900 Mm^3 , 280 Mm^3 , 0.7 and 0.5 respectively. Generate a sequence of 3 annual flows assuming that the flows are normally distributed and taking the chain of 3 standard normal random numbers as 1.45, -1.16 and 0.087. Use AR(2) model. [12]
6. a) Explain the Yevjevich and Quimpo model for generating daily flows. [8]
 - b) The first and second serial correlation coefficients of observed annual rainfall are 0.7 and 0.5 respectively. Assuming ARMA (1,1) model is a good fit, find the parameters of the model. [6]
7. Describe the methods for the generation of uniformly distributed random numbers. [8]

Exam.	Regular		
Level	BE	Full Marks	80
Programme	BCE	Pass Marks	32
Year / Part	IV / II	Time	3 hrs.

Subject: - Time Series Analysis (Elective III) (CE78505)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ The figures in the margin indicate Full Marks.
- ✓ Assume suitable data if necessary.

1. Explain the following terms: deterministic process, white noise, marginal distribution, parsimony of parameters [4x2]

2. Define the univariate and multivariate time series and stationary and non-stationary time series data. [2+2]

3. The duration of a monsoon storm X is a random variable whose probability density function is given by [2+2+2]

$$f(x) = kx^2 \text{ for } 0 \leq x \leq 6h$$

$$= k(12-x)^2 \text{ for } 6h \leq x \leq 12h$$

$$= 0 \text{ elsewhere}$$

Find the value of k. What is the probability that the duration of the storm is between 6h to 9h. What is the probability that the duration of the storm is between 6h to 9h, given that the storm has lasted from 6h?

4. (a) Define central limit theorem. How is this concept useful for the analysis of hydrological time series data? [2+2]

(b) The annual runoff of a stream is modeled by a normal distribution, with mean = 4100 m³/s and standard deviation = 750 m³/s. Find the ranges which is symmetrical about mean within which the runoff will lie with a probability of 0.5. Also, compare the range using Chebyshev's inequality concept and comment on the result.

For normal distribution, take Z = 0.675 for F(Z) = 0.75 and Z = -0.675 for F(Z) = 0.25. [4+4]

5. (a) What is autocorrelation analysis? How is it useful for the investigation of the time series data? [2+6]

b) Obtain the parameters of AR(2) model from Yule-Walker equation. Also, discuss the condition of stationarity and the nature of autocorrelation function of AR(2) model with sketches. [2+2+4]

6. (a) Show that the autocorrelation function for MA(m) model is

$$\rho_k = \frac{(-\beta_k + \beta_1\beta_{k+1} + \beta_2\beta_{k+2} + \dots + \beta_{m-k}\beta_m)}{(1 + \beta_1^2 + \beta_2^2 + \dots + \beta_m^2)}$$

Where ρ_k = autocorrelation coefficient for lag k, m = order of model and β = parameter of model. [8]

7. a) Explain the Thomas-Fiering model for generating seasonal flows [8]

b) Assuming an ARMA(1,1) model is a good fit to describe the normally distributed annual flows of a stream whose mean, standard deviation, first and second serial correlation coefficients of observed annual flows of a stream are estimated to be 1210 Mm³, 570Mm³, 0.535 and 0.463 respectively. Generate a sequence of 3 annual flows using the following independent standard normal random numbers 1.123, -0.821 and -0.342. [10]

8. Generate a chain of 6 uniformly distributed random numbers in the range 0 to 0.999 using the mixed congruential generator. Take multiplier = 361, increment = 56377, modulus = 10⁵ and seed = 42765. [8]

Exam.	Regular / Back		
Level	BE	Full Marks	80
Programme	BCE	Pass Marks	32
Year / Part	IV / II	Time	3 hrs.

Subject: - Time Series Analysis (Elective III) (CE78505)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
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1. Is there any linkage of time series model to physical process in Hydrology? Explain by giving example of rainfall-runoff process. [8]
2. What do you mean by persistency in a time series? Differentiate the following: [2+2+2+2]
 - a) Stationary and non-stationary time series
 - b) Univariate and multivariate time series
 - c) Trend and peridodicity
3. a) The probability density function of normal distribution is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$
 Find the parameters μ and σ by method of maximum likelihood. [6]
 - b) The probability density function of a random variable is given by

$$f(x) = 6x(1-x) \text{ for } 0 < x < 1$$

$$= 0 \text{ elsewhere}$$
 Find the probability that x will take on a value within 1.56 times standard deviations of the mean and compare it with the lower bound provided by Chebyshev's inequality.
4. a) Mean daily discharge (m^3/s) of a river for 10 days is given below: [8]

50, 48, 49, 67, 78, 100, 115, 300, 650, 400

 Compute first autocorrelation coefficient using the simplified form of equation.
 - b) For MA (2) process, following data are given: $r_1 = 0.4$, $r_2 = 0.19$. Compute parameters β_1 and β_2 . [4]
5. a) Starting form Yule-Walker equation for AR (m) model, obtain the parameters of AR(1) and AR(2) model. Also explain the nature of autocorrelation function of AR(1) model with sketches. [2+2+3]
 - b) Assuming an ARMA(1,1) model is a good fit to describe the normally distributed annual flows of a stream whose mean, standard deviation, first and second serial correlation coefficients of observed annual flows of a stream are estimated to be 75 m^3/s , 3 m^3/s , 0.5 and 0.3 respectively. Generate 3 annual flows. The chain of 3 standard normal random numbers may be taken as -1.15, 0.38 and 1.20. [10]
 - c) What is an ARIMA model. [3]
6. a) Explain the Thomas-Fiering model for generating seasonal flows. [10]
 - b) What is the difference between correlogram and periodogram? [2]
7. Generate a chain of 6 uniformly distributed random numbers in the range 0 to 0.999 using the mixed congruential generator. Take multiplier = 361, increment = 56377, modulus = 10^5 and seed = 42765. [8]

Exam.	New Back (2066 & Later Batch)		
Level	BE	Full Marks	80
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Subject: - Time Series Analysis (Elective III) (CE78505)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
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- ✓ Assume suitable data if necessary.

1. Differentiate between deterministic and random process. [3]
2. What is time series model? What are the processes that have to be followed in time series modeling? [1+2]
3. The annual maximum discharge (m^3/s) of a river for a period of 7 years is given below: [8]
14, 16, 13, 18, 26, 22, 19

Test the series for the presence of the trend using linear regression technique.

4. a) The probability density function of a random variable is given by $f(x) = \lambda e^{-\lambda x}$ for $x > 0$. Find the parameter λ of the distribution by (a) method of moments and (b) method of maximum likelihood. [3+3]
- b) The probability density function of a random variable is given by [6]
 $f(x) = 6x(1-x)$ for $0 < x < 1$
 $= 0$ elsewhere

Find the probability that x will take on a value within 1.56 times standard deviations of the mean and compare it with the lower bound provided by Chebyshev's inequality.

5. a) What are different types of periodic series? Describe each of them in brief. [4]
- b) Derive the Yule-Walker equation for AR(m) model and obtain the relationship between the variance of AR(m) process and independent process. [6+4]
- c) Classify the following process as AR, MA or ARMA. Verify whether they are stationary. $(X_t - \mu) = (X_{t-1} - \mu) + e_t + 0.2e_{t-1}$ [1+2]
6. Generate synthetic seasonal flows for 4 seasons for a year from the following statistical parameters using Thomas-Fiering model. [12]

Season	Mean flow (m^3/s)	Standard deviation	Correlation with previous season	$Z_t: N(0,1)$
1	11	8.8	0.63	0.25
2	20	10	0.62	-1.71
3	58	57	0.20	0.31
4	10	8	0.05	0.11

7. The first 9 autocorrelation coefficients computed from 150 years of annual flow record at a river site are: 0.46, 0.005, -0.09, -0.06, -0.05, -0.01, -0.02, -0.016, -0.06 [8+1]
Test the flow series for independence at 95% confidence level. Also suggest a suitable model for the series.
8. a) Starting from the autocorrelation function of MA (m) model, obtain the parameters of MA (1) and MA (2) model. [8]
- b) Generate a chain of 5 uniformly distributed random numbers in the range 0 to 0.999 using the mixed congruential generator. Take multiplier = 321, increment = 50001, modulus = 10^5 and seed = 38975. [8]